Journal of Computational Physics **148,** 299–301 (1999)
Article ID jcph.1998.6139, available online at http://www.idealibrary.com on IDEAL®

ERRATUM

Volume 136, No. 1 (1997) in the article "Cauchy—Characteristic Evolution and Waveforms," by Nigel T. Bishop, Roberto Gómez, Paulo R. Holvorcem, Richard A. Matzner, Philippos Papadopoulos, and Jeffrey Winicour, pages 140–167: On page 148, Eq. (3.3), where one reads $\partial x \phi$, one should read $\partial_x \phi$. On page 160, where one reads "Furthermore, the extrapolated maching solution $\phi_{1-\mathrm{D}}^{\mathrm{C_1F}}$ (obtained from the solutions with $R_m = 234/17$) coincides...," one should read "Furthermore, the extrapolated matching solution $\phi_{1-\mathrm{D}}^{\mathrm{C_1N}}$ (obtained from the solutions with $R_m = 182/17$) coincides...."

Also, the following figure captions were omitted:

FIG. 1. Initial Cauchy data are evolved from t_0 to time t_1 throughout the region D_{1^-}. Characteristic data induced on C_{1^-}, combined with the initial characteristic data on C_{0^+} are used to evolve the region D_{1^+}. This produces Cauchy data at time t_1 in the region $r \leq R_m$. Similarly, Cauchy evolution is used in the region D_{2^-}, bounded on the right by C_{2^-}. The characteristic data induced on C_{2^-}, together with those on C_{1^+}, are sufficient to evolve through the region D_{2^+}. The process can be iterated to carry out the entire future evolution of the system.

FIG. 2. Cauchy grid points are indicated by squares and characteristic grid points by circles. The triangles indicate points E and F, where the time level t_n intersects the retarded time levels u_{n-1} and u_{n-2}. Initial data are given at the shaded points. Evolution proceeds iteratively by determining field values at the unshaded points. The matching scheme provides boundary values at C $(r = R_m)$ and D $(r = R_B + h)$ for the characteristic and Cauchy grids, respectively.

FIG. 3. Parallelogram formed by incoming and outgoing characteristics which intersect at vertices P, Q, R, S. PQ is taken to be at retarded time level u_n and RS at level u_{n-1}. By using the identity (2.5), the field value g_Q can be obtained in terms of already known values of $g(r, u_n)$ $(r \leq r_P)$ and $g(r, u_{n-1})$.

FIG. 4. The line AF, along which we interpolate between the Cartesian and spherical grids, shown schematically. B is an interior boundary point, D is a nearest neighbor of B which is an exterior point. A is the nearest neighbor to B opposite to D, and by construction, an interior point. E and F lie on the line AB, on two previously computed characteristic cones, and C is a point on the first spherical shell of the characteristic grid.

FIG. 5. For some exceptional points, the nearest neighbor D can be reached along two coordinate lines of the Cartesian grid starting from two distinct interior boundary points, B and B'.

FIG. 6. Grid points in the (r, t) plane for the characteristic evolution used in the matching schemes of Section III.

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FIG. 7. Results of stability tests for the matching algorithm I. The parameters α and β defined by (4.3) were independently varied, and ρ, K were chosen according to (4.1)–(4.2) and (2.12), respectively. The Cartesian grid size was fixed at $M = 22$. Stable algorithms are obtained when $\beta \geq 1$ and $\alpha \approx 1.25$. The dashed curves are contours of constant Courant number ρ (from left to right, the contours shown have $\rho = 0.236$, 0.151, 0.101, 0.077, and 0.062). Stable and unstable algorithms are indicated by solid and open circles, respectively.

FIG. 8. Results of stability tests for the matching algorithms C_1 (a) and C_2 (b). In this set of experiments, R_m, N, h, N_g, N_L, N_B have the fixed values $\frac{182}{17}$, 21, $\frac{26}{51}$, 20, 31, 125, respectively, while the remaining parameters are chosen so that $h_{ang}/h_{rad} \simeq \Delta/R_m\Delta\xi \simeq 2.1$, $s_g/h_{ang} \simeq s_g/R_m\Delta \simeq 3.7$, $s_L/\Delta \simeq 14$, $s_B/h = 8$. The independent variables in this study are $\rho = \Delta t/h$ and $\alpha' = R_m\Delta/h$. The dashed horizontal line represents the upper limit on ρ imposed by the CFL condition for the Cauchy evolution ($\rho \leq 3^{-1/2}$), while the dashed line through the origin represents the analogous limit for the characteristic evolution. Stable and unstable algorithms are indicated by solid and open circles, respectively.

FIG. 9. Comparison between different algorithms for the solution of the forced linear wave equation: S_1 (a), C_1 (b), S_2 (c), C_2 (d), K (e), and I (f). The forcing consists of a superposition of four spherical harmonic sources located at various positions in the interior computational grid (see text for details). The interface between the interior and exterior grids is located at $r = \frac{182}{17}$. In (a)–(d), the maximum errors at the grid points (4.5)–(4.6) are shown for the selected times (4.4), using the discretizations $N_{4/3}$ (open triangles), N_1 (open squares), $N_{4/5}$ (open circles), $N_{2/3}$ (solid triangles), and $N_{1/2}$ (solid circles). In (e), the maximum errors are shown for the same selected times and grid points using the discretizations $N''_{4/3}$ (open triangles) and N''_1 (open squares). In (f), the maximum errors at the grid points (4.9)–(4.10) are shown for the selected times (4.4), using the discretizations $N'_{4/3}$ (open triangles), $N'_{4/5}$ (open circles), and $N'_{4/7}$ (solid squares). In each case, the errors resulting from applying Richardson extrapolation to the results for the finest grids ($N_{2/3}$ and $N_{1/2}$, or $N'_{4/5}$ and $N'_{4/7}$, or $N''_{4/3}$ and N''_1) are indicated by star symbols. An analogous set of tests using the discretizations with interface at $r = \frac{234}{17}$ (F_λ, F'_λ, and F''_λ) yields similar results for all the above algorithms. For comparison, the maximum value of $|\phi|$ during the evolution is about 1.4.

FIG. 10. Errors at radiative infinity resulting from three matching schemes for the solution of the forced linear wave equation: C_1 (a), C_2 (b), and I (c). The forcing and the discretizations employed are the same as in Fig. 9; the numerical solutions with different resolutions are also indicated by the same symbols. Maximum errors are shown for the selected characteristic cones (4.7), and the errors resulting from applying Richardson extrapolation to the results for the finest grids ($N_{2/3}$ and $N_{1/2}$ or $N'_{4/5}$ and $N'_{4/7}$) are indicated by star symbols. In (a)–(b), since the numerical solutions of different resolutions utilize distinct triangulations of the unit sphere, Richardson extrapolation had to be preceded by an interpolation into a common set of angular directions (this was chosen as the triangulation used in discretization $N_{4/3}$). The interpolation method had error $O(\Delta^4)$ and should not introduce a significant loss of precision. In (c), the extrapolation technique was applied at the angular directions with stereographic coordinates $p = k/8$, $q = l/8$ ($k, l = -8, -7, \ldots, 8$), which are common to discretizations $N'_{4/5}$ and $N'_{4/7}$. An analogous set of tests using the discretizations with interface at $r = \frac{234}{17}$ (F_λ and F'_λ) yields similar results for all the above algorithms. For comparison, the maximum value of $|g|$ during the evolution is about 2.5.

FIG. 11. Comparison between different one-dimensional algorithms for the solution of the forced nonlinear wave equation (4.15). The forcing has spherical symmetry about the origin (see text for details). The data shown in the figure correspond to $\|\phi^{C_1F}_{1-D} - \phi_{1-D}\|_2$

(stars), $\|\phi^{C_1N}_{1-D} - \phi^{C_1F}_{1-D}\|_2$ (solid circles), and $\|\phi^{S_2F}_{1-D} - \phi_{1-D}\|_2$ (open triangles). The notation for the various numerical solutions is explained in the text. The notation $\|\cdot\|_2$ denotes the r.m.s. value over the grid points (4.5)–(4.6) (in order to facilitate comparisons with the three-dimensional runs, each one-dimensional solution was interpolated onto these selected grid points, and the r.m.s. value computed as a three-dimensional average). For comparison, the r.m.s. value of ϕ is between 0.04 and 0.16 during most of the evolution.

FIG. 12. Comparison between different three-dimensional algorithms for the solution of the forced nonlinear wave equation (4.15): S_1 (a), C_1 (b), S_2 (c), C_2 (d), K (e), and I (f). The forcing has spherical symmetry about the origin (see text for details), and the discretizations are the same as in Fig. 9. In (a)–(e), the r.m.s. errors over the grid points (4.5)–(4.6) are shown for the selected times (4.4). In (f), the r.m.s. errors are evaluated over the grid points (4.9)–(4.10) for the same selected times. The exact solution is approximated to high accuracy by a numerical solution ϕ_{1-D} produced by a one-dimensional finite-difference code which solves the spherically symmetric version of (4.15). For comparison, the r.m.s. value of ϕ is between 0.04 and 0.16 during most of the evolution.

FIG. 13. Sensitivity of the numerical solutions of (4.15) by different algorithms with respect to changes in the radius of the interior computational grid. The forcing distribution has its support on two ellipsoids (see text for details). Using the notation for extrapolated solutions defined at the end of Section IV.B, the data shown in the figure correspond to $\|\phi^{S_1N}_{2/3,1/2} - \phi^{S_1F}_{4/5,2/3}\|_2$ (open triangles), $\|\phi^{S_2N}_{2/3,1/2} - \phi^{S_2F}_{4/5,2/3}\|_2$ (open squares), $\|\phi^{KN''}_{4/3,1} - \phi^{KF''}_{2,4/3}\|_2$ (open circles), $\|\phi^{C_1N}_{2/3,1/2} - \phi^{C_1F}_{4/5,2/3}\|_2$ (solid triangles), $\|\phi^{C_2N}_{2/3,1/2} - \phi^{C_2F}_{4/5,2/3}\|_2$ (solid squares), and $\|\phi^{IN'}_{4/5,4/7} - \phi^{IF'}_{4/3,4/5}\|_2$ (solid circles). The r.m.s. differences are evaluated over either the grid points (4.5)–(4.6) (for algorithms C_1, C_2, S_1, S_2, and K) or the grid points (4.9)–(4.10) (for algorithms I), at the selected times (4.4). For comparison, the r.m.s. value of ϕ is between 0.04 and 0.08 during most of the evolution.

FIG. 14. Sensitivity of the numerical solutions of (4.15) by different algorithms with respect to changes in the order of the boundary conditions. The forcing distribution has its support on two ellipsoids (see text for details). Using the notation for extrapolated solutions defined at the end of Section IV.B, the data shown in the figure correspond to $\|\phi^{S_1N}_{2/3,1/2} - \phi^{S_2N}_{2/3,1/2}\|_2$ (open triangles), $\|\phi^{S_1F}_{4/5,2/3} - \phi^{S_2F}_{4/5,2/3}\|_2$ (open squares), $\|\phi^{C_1N}_{2/3,1/2} - \phi^{C_2N}_{2/3,1/2}\|_2$ (solid triangles), and $\|\phi^{C_1F}_{4/5,2/3} - \phi^{C_2F}_{4/5,2/3}\|_2$ (solid squares). The r.m.s. differences are evaluated over the grid points (4.5)–(4.6), at the selected times (4.4). For comparison, the r.m.s. value of ϕ is between 0.04 and 0.08 during most of the evolution.

FIG. 15. Errors in the evaluation of the Gaussian function (B14) at $x = y = z = \frac{1}{2}$ by several multiquadric-based approximations. The curves labeled $p = -1$ and $p = 0$ correspond respectively to the uncorrected multiquadric approximation and the interpolation formula with normalized coefficients $\hat{c}_\mu$. The remaining curves represent the errors in corrected multiquadric interpolation formulas with error $O(\lambda^{p+1})$, $p = 1, 2, 3, 5$. The ratio $\max|\delta_\mu|/\max|\hat{c}_\mu|$, which measures the size of the corrections, is smaller than 0.044 for all the above corrected approximations.

FIG. 16. Errors in the evaluation of the second derivative of the Gaussian function (B14) with respect to x at $x = y = z = \frac{1}{2}$ by several multiquadric-based approximations. The curve labeled $p = -1$ corresponds to the uncorrected multiquadric approximation, and the remaining curves represent the errors in corrected multiquadric finite-difference formulas with error $O(\lambda^{p-1})$, $p = 0, 1, 2, 3, 5$. The ratio $\max|\delta_\mu|/\max|\tilde{c}_\mu|$, which measures the size of the corrections, is smaller than 0.097 for all the above corrected approximations.

This erratum is Article No. jcph.1998.6139.